SCALYBOT: A SNAKE-INSPIRED ROBOT WITH ACTIVE CONTROL OF FRICTION

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ABSTRACT

Snakes are one of the world's most versatile locomotors, at ease slithering through rubble or ratcheting up vertical tree trunks. Their adaptations for movement across complex dry terrain thus serve naturally as inspirations for search-and-rescue robotics. In this combined experimental and theoretical study, we perform experiments on inclined surfaces to show that a snake's scales are critical anatomical features that enable climbing. We find that corn snakes can actively change their scales' angles of attack by contracting their ventral muscles and lifting their bodies. We use this novel paradigm to design Scalybot, a two-link limbless robot with individually controlled sets of belly scales. The robot can ascend styrofoam plates inclined up to 45°, demonstrating a climbing ability comparable to that of a corn snake in the same conditions. The robot uses individual servos to provide a spatial and temporal dependence of its belly friction, effectively anchoring the stationary part of its body while reducing frictional drag of its sliding section. The ability to actively modulate friction increases both the robot's efficiency over horizontal surfaces and the limiting angles of inclination it can ascend.

NOMENCLATURE

m Mass of Scalybot
g Gravitational acceleration
L Body length
L_{min}, L_{max} Minimum and maximum length during one period
ΔL periodic change in body length
τ Period of motion
μ_f, μ_b Forward and backward dynamic friction coefficients
μ_f^s, μ_b^s Forward and backward static friction coefficients
$l, \dot{l}$ Inter-nodal distance and velocity
f Internal force
$x_i, \dot{x}_i, \ddot{x}_i$ Displacement, velocity and acceleration of ith mass
$\bar{V}, \ddot{\bar{x}}$ Velocity and acceleration of center of mass
θ Inclination angle versus ground
α Scales' angle of attack
Fr Froude number

INTRODUCTION

Designing an all-terrain robot is a challenging task that has drawn the attention of roboticists, biologists and applied mathematicians. Such a robot has a variety of applications, from interplanetary exploration, exploration within the human body as in "robotic colonoscopy" and search-and-rescue missions beneath the rubble of collapsed buildings. One challenge for such robots is overcoming slopes of varying inclination. Steeper slopes are more difficult to climb because of the reduction in friction force with the underlying surface and the consequences of losing one's grip and sliding down the slope.

Previously built bio-inspired climbing robots rely upon successful functionalities observed in their biological counterparts. For climbing smooth surfaces like glass, Geckobot [1] relies upon suction cups and Stickybot [2] upon directional dry adhesives such as found in gecko feet. For climbing rougher surfaces like brick, Spinybot [3, 4] uses spines to dig into asperities. The range of such legged robots is limited by their inability to cross large obstacles in their path, move through crevices smaller than their body width and transition from horizontal to vertical surfaces. Overcoming these limitations will require a climbing robot that marries anchoring abilities with snake-like flexibility. This ability is clearly observed in certain snakes, which are known to climb up trees as shown in Fig.1a.

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The versatility of snakes across a range of topography, inclination and surface textures has drawn much interest over the years towards building snake-like robots. Reviews of snake-like robots are presented by Hirose and Hopkins [5, 6], which categorize snake-like robots into those with free-rolling or motorized wheels, those with motorized tank treads, or extensible bodies relying upon vertical waves or linear expansions [6]. Wheels provide the snake-like robots with a forward-transverse frictional anisotropy associated with the wheel's relative ease of rolling forward compared to sliding sideways. However, a reliance on wheels prevents most robots from climbing slopes (with an exception provided by Choset [7], whose robot can climb poles and vertical channels).

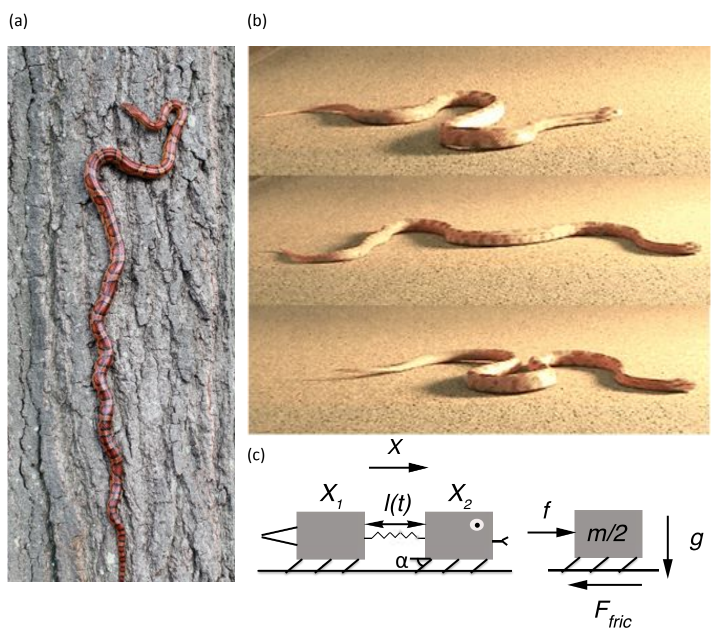

Figure 1. (a) A corn snake climbing a tree. (b) The image sequence of the concertina locomotion of a corn snake. (c) The corresponding 2-mass model describing Scalybot's dynamics.

Few attempts have been made to design artificial snake scales to aid the locomotion of snake-like robots uphill. Dowling [8] used plain spandex, sequins, and polyethylene braids for providing purchase in his snake-like robot. Recent work indicates that such an approach may be potentially valuable to robotics: In experiments of live snakes over flat surfaces, Hu *et al.* [9] found the scales of snakes provide a frictional anisotropy that aids slithering locomotion over flat surfaces.

The goal of the current study is to determine if this paradigm of locomotion-via-scales has potential for improving the climbing ability of snake-like robots. We begin by reporting our methods for robot construction, friction measurement and testing. We proceed with a theoretical model for the robot's locomotion. Numerical results of this model are then presented along with our experimental measurements of the robot's motion. We close with a discussion of the limitations of our model, an evaluation of the robot's performance and suggestions for its improvement and incorporation of its design into other snake-like robots.

METHODS

Building Scalybot

We have designed our robot to mimic a "concertina" mode of locomotion in which the body is sequentially extended and contracted, as shown by a corn snake in Fig.1b. The simple kinematics of concertina motion is similar to that of inchworms or earthworms, which can be crudely considered as having one degree of freedom, the length of their bodies. Propulsion consists of two phases. In the first, tail is anchored while the head is pushed forward (Fig.1b). In the second, the head is anchored while the tail is pulled forward. This "ratcheting" is fundamentally a slow process because of the loss of body inertia due to decelerating and anchoring each part of the body. The simplicity of the associated kinematics, however, will allow us to highlight the importance of the belly scales during propulsion.

We based our robot's scales on those of a corn snake. A snake's belly scales resemble the overlapping shingles of a house. This geometry provides the snake with a preferred direction of sliding: the scales slide easily over surfaces when the snake slithers forward (Fig.2a), but dig in when the snake is gently pulled by its tail (Fig.2b). The friction anisotropy of dead snakes was first reported by Gray and Lissman [10]. They measured dynamic friction coefficients of grass snakes on several materials. Based on their experiments the friction anisotropy (ratio of backward to forward friction coefficients) on dry metal is 1 and on rough sand paper is 4.8. Although not obviously critical on horizontal surfaces, we shall see in our experiments that this level of anisotropy is useful for climbing.

Based on snake concertina kinematics and snake scale design, we have built Scalybot, a simple extensional robot that can control the angle of attack of its scales to modify its resistance to sliding in different directions. The robot is made of 2 similar segments (Fig.3). The segments are each housed in a steel casing and connected to each other by an SMC pneumatic actuator. Two 24vDC solenoid valves control the pneumatic cylinder position. Two manual flow control valves and an inline miniature air regulator are used to control air flow rate and pressure, respectively. A household ventilation register was modified to manufacture the scales. Each body segment contains 5 steel scales arranged as louvers whose pitch was varied by a linkage system connected to a servo motor (HS-311 Hitec). Both solenoid valves and servos are controlled using an Arduino UNO microcontroller board which is programmed using Arduino software. The total weight of the robot is 1.16 kg, which does not include the sources of pneumatic pressure or electric energy.

Friction Measurements

Friction coefficients were measured by placing the robot in two orientations on the plane (facing up the slope correspond-

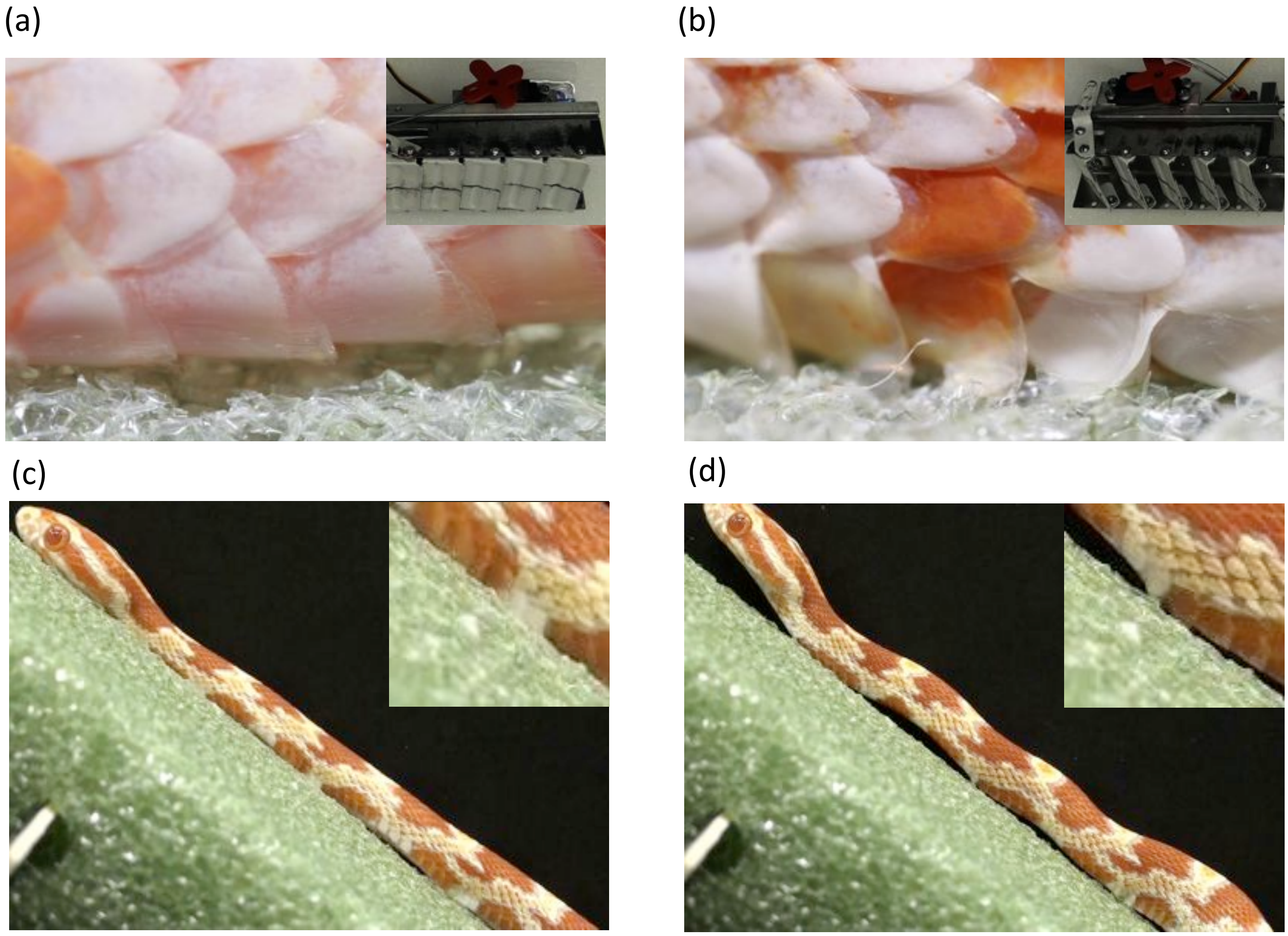

Figure 2. (a,c) Corn snake and Scalybot scales at their minimum angles of attack (flat), and (b,d) at maximum angles of attack. Snakes can modify their scales' angles of attack to increase their friction anisotropies. Scalybot uses the same concept to climb steep slopes, as shown by the insets.

ing to use of the backward friction coefficient). Static friction coefficients μ^s were given by $\tan\theta$ where θ is the minimum incline angle at which the robot begins to slide. Dynamic friction coefficients μ were measured by filming the robot sliding down an incline of angle ϕ and measuring its displacement x and duration of sliding t. The dynamic friction coefficient can then be estimated using the implicit relation $x = \frac{1}{2}g(\sin\phi - \mu\,\cos\phi)t^2$.

Friction measurements of our robot and three corn snakes were taken while on open-cell rigid styrofoam. This material was chosen because its roughness of 1.2 mm was greater than both the robot and corn snake scale thickness (0.8 mm and 45 μm, respectively). In this regime, friction coefficients are significantly affected by scale angles of attack, in comparison to on smoother surfaces such as a tabletop with a roughness of 20 μm.

MODEL

The speed, $\bar{V}$, of our robot was predicted using the following simple model of its dynamics, whose schematic is shown in Fig.1c. We partition the device into 2 nodes representing point-masses of mass $m/2$, where m is the total mass of Scalybot. These nodes, labelled i = 1 or 2, are separated by an inter-nodal distance $l(t)$. The robot can adjust the relative position of its nodes by applying internal forces f using its pneumatic piston. Resisting motion of the masses are inertia and dynamic frictional forces along the ground. The dynamic friction coefficients of the belly sliding in the forward and backward directions are μ_f and μ_b respectively. An important assumption is our neglect of static friction, whose consequences are given in the Performance section. Newton's second law applied to each of the nodes yields:

$$\begin{aligned}\ddot{x}_1 &= g\cos\theta[-\mu_f H(\dot{x}_1) + \mu_b H(-\dot{x}_1)] - g\sin\theta + \frac{2}{m}f \\ \ddot{x}_2 &= g\cos\theta[-\mu_f H(\dot{x}_2) + \mu_b H(-\dot{x}_2)] - g\sin\theta - \frac{2}{m}f, \qquad (1)\end{aligned}$$

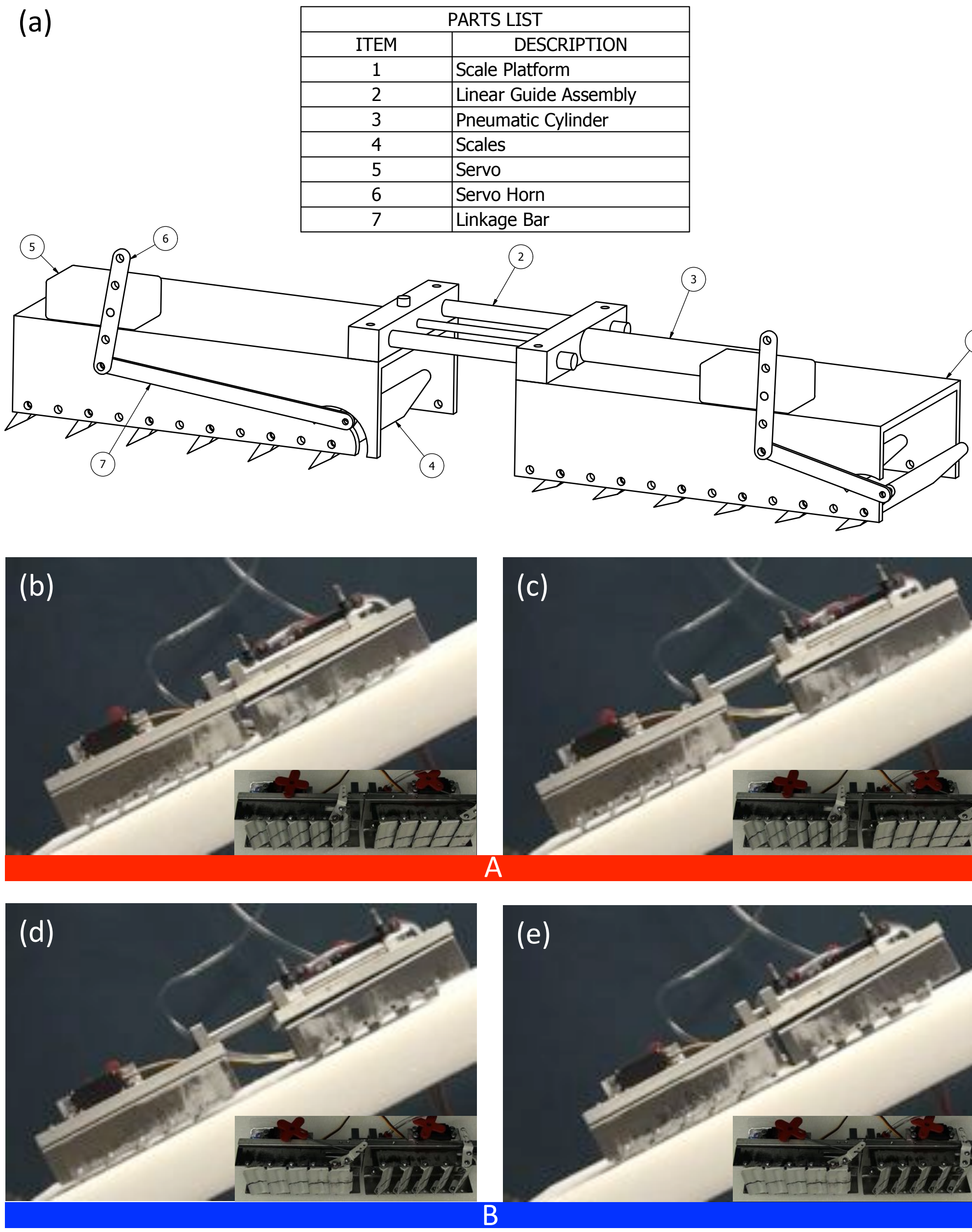

Figure 3. (a) CAD drawing of Scalybot and list of item descriptions. (b-e) The image sequence for one period of motion of Scalybot. Insets show oblique views of the belly scales for the stationary and moving segments. Phases A and B denote kinematic phases defined in Fig.4.

where θ is the substrate inclination angle, x_i is the position of the ith mass and $H(x) = \frac{1}{2}(1 + sgn(x))$ is the Heaviside step function. Sum of the equations in (1) yields the center-of-mass acceleration, $\ddot{\bar{x}}$:

$$\ddot{\bar{x}} = \frac{g\cos\theta}{2}\left[-\mu_f \sum_{i=1}^{2} H(\dot{x}_i) + \mu_b \sum_{i=1}^{2} H(-\dot{x}_i)\right] - g\sin\theta. \qquad (2)$$

Non-dimensionalizing Eq. (2) using the Scalybot length L and its period of motion τ, we obtain

$$Fr\ddot{\bar{x}} = \frac{\cos\theta}{2}\left[-\mu_f \sum_{i=1}^{2} H(\dot{x}_i) + \mu_b \sum_{i=1}^{2} H(-\dot{x}_i)\right] - \sin\theta \qquad (3)$$

where the dimensionless parameter is

$$Fr = \frac{\text{inertia}}{\text{gravity}} = \frac{L}{\tau^2 g}\,. \tag{4}$$

In our experiments, the Froude number is $Fr \sim 0.39-1.77$ over the range of $\dot{l}$ prescribed. A small Froude number indicates the inertial is small compared to gravitational forces.

Numerical integration of Eq. (3) allows us to determine steady state behavior given prescribed kinematics. The robot's kinematics is given by the inter-nodal distance, $l(t)$. This function is characterized by three parameters, L_{min}, ΔL, and τ, as shown in Fig.4. An explicit Runge-Kutta (4,5) formula, the Dormand-Prince pair, is used to solve Eq. (3) numerically in MATLAB [11]. Using the state-space form of Eq. (3), acceleration, velocity and position of center of mass are calculated. We characterize the effectiveness of the robot's motion by two parameters, its steady center of mass speed, $\bar{V}$, and the steepest incline it can climb of our prescribed test surface. Speed is non-dimensionalized according to the robot's length L and period τ.

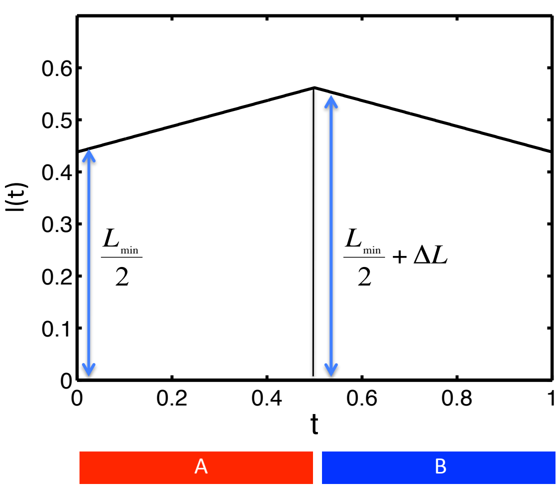

Figure 4. The time course of inter-nodal spacing $l(t)$. There are two phases in this function: In phase A (extension), the head is pushed forward; in phase B (contraction) the tail is pulled forward. The stationary segment uses active friction changes to provide anchorage.

RESULTS

We report upon the frictional properties and performance of Scalybot and corn snake.

Friction Measurements

To ascend inclined planes, we observed snakes performing a combination of rectilinear and concertina gaits. In rectilinear motion, snakes exhibited a slow creeping of their bellies and lifting of their scales. Snakes exhibited clear adaptations to prevent falling down the incline. On surfaces inclined greater than 20°, snakes used a form of "emergency braking" to prevent from sliding backwards. Fig.2c-d shows a snake performing concertina up a 35° incline. When it begins to lose its grip, the snake freezes its body in an S-shaped configuration. This limits the snake's ventral contact with the ground to a few discrete points where scales appear to be catching as shown in insets of Fig.2c-d. After the snake has circumvented sliding and regained its grip, it resumes moving up the incline.

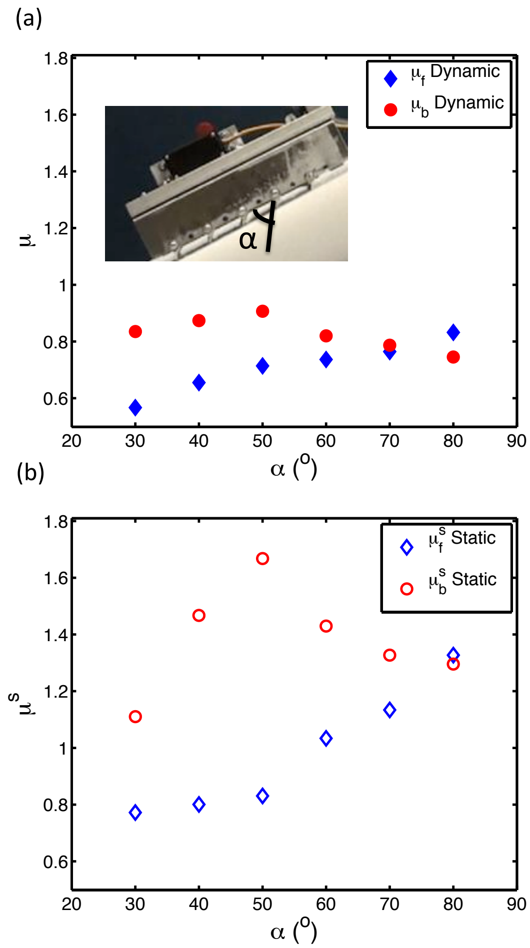

Figure 5. Effect of Scalybot scales' angles of attack, α, on (a) dynamic friction coefficients, and (b) static friction coefficients on Styrofoam. Both forward and backward directions are measured. For obtaining maximum available friction anisotropy, Scalybot sets the scales of its stationary segment at 50° and those of its moving segment at 30°. The friction anisotropy μ_b/μ_f is 1.6 for dynamic friction coefficients and 2.16 for static ones.

The braking mechanism we observed suggests that behavior is important in modifying the snake's friction. This hypothesis is consistent with our measurements of corn snake friction coefficients. Conscious snakes have greater dynamic friction anisotropies (μ_b/μ_f) of 1.65±0.25 (N=3) than unconscious snakes (1.55±0.15, (N=3)).

To obtain insight into the mechanism of friction modification, we conducted experiments using snake-scale mimics constructed of 0.8-mm thick steel sheets. These sheets were later used as the scales of our robot. We characterize the anchoring ability of a scale according to its angle of attack α with respect to the body (Fig.1c). During locomotion, the angle of attack with respect to the underlying substrate may slightly exceed α as a result of small inclinations (3°) of the robot relative to the substrate due to the combined effects of scale lifting and piston flexibility.

As shown in Fig.5, the backwards friction is highly dependent on a scale's angle of attack, α. Specifically, there exists an optimal angle ($\alpha = 50°$) that best resists sliding backwards. This optimal angle is comparable to the scale angle of attack (20-30°) observed for live snakes as they are gently pulled by their tails (Fig.2a-b). When snakes wish to progress forward, they flatten their scales to 0° to reduce frictional sliding. Corn snake have a minimum scale angle of zero degrees, in part because their scales and body are flexible. However, since we manufactured the robot's scales to be both rigid and overlapping, the minimum angle for Scalybot's scales is 30°, which will be referred henceforth as the scales' default "flat" orientation.

To maximize friction anisotropy, Scalybot sets the scales of its stationary segment to 50° and those of its sliding section to 30°. At this setting dynamic friction anisotropy μ_b/μ_f is 1.6 and static friction anisotropy is 2.16. This is substantially greater than the friction anisotropies for scales kept flat at 30° (1.47 and 1.44, respectively). The activation of the scales thus clearly improves friction anisotropy. Moreover, by activating its scales, Scalybot has similar dynamic and static friction anisotropies to conscious corn snakes on Styrofoam ($\mu_b/\mu_f = 1.6$ for dynamic and $\mu_b^s/\mu_f^s = 1.76$ for static), further suggesting we have done well in optimizing the frictional properties of our robot.

Kinematics

In analogy to a snake's concertina motion (Fig.1b), we prescribed the kinematics of the robot using the function $l(t)$, shown by the triangular waveform in Fig.4. The robot has two phases of motion (A-B) during its period τ: these consist of expansion and contraction phases at constant inter-nodal velocities $\dot{l}$ and $-\dot{l}$, respectively. The minimum and maximum lengths of the robot are L_{min} and $L_{max} = L_{min} + \Delta L$, respectively. During testing, we varied the speed of expansion and contraction by keeping ΔL constant and adjusting the period τ.

The kinematics of the scales during concertina was chosen to best prevent backwards sliding of the anchoring mass. In phase A, the tail is stationary while the head moves forward; in phase B, the segments reverse roles. During these phases, moving and stationary segments have scale angles of 30° and 50°, respectively.

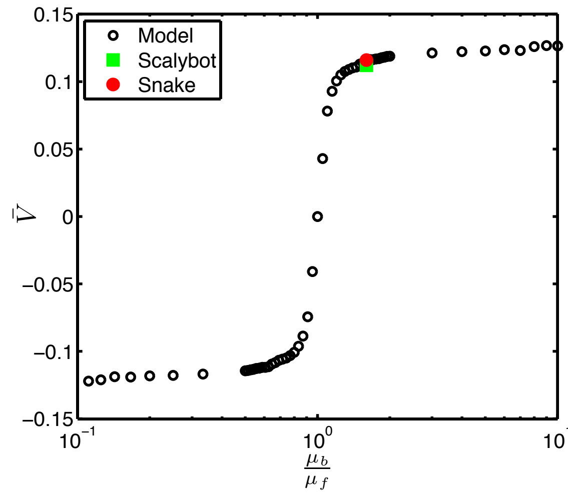

Figure 6. Numerical predictions of the velocity of center of mass, $\bar{V}$, as a function of friction anisotropy μ_b/μ_f, predicted by Eq. (3); Scalybot and corn snake velocities are included for comparison. Scalybot uses active anisotropy by changing its scales' angles of attack so that $\mu_b/\mu_f = 1.6$.

Performance

In terms of maximum climbing angle, our robot showed a similar performance to that of a corn snake. Using active scales, we observed that our robot was capable of climbing up 45° slopes and remaining at rest on slopes of 60°. This is comparable to the climbing performance of a corn snake, which on the same material can climb at 35° and remain at rest at 43°.

Fig.6 shows the predicted steady velocity as a function of dynamic friction anisotropy on a horizontal surface. With activated scales and a contraction speed of $\dot{l} = 6$, Scalybot is able to move forward at a speed of 0.112, well-predicted by our model. This speed is similar to that of a corn snake (0.116) on the same material in a channel of width 2cm. Moreover, the predicted speeds are near the maximum speeds for such motion, as shown by the asymptotic behavior of $\bar{V}$ at higher anisotropies (Fig.6). This correspondence suggests we have reached an ideal frictional anisotropy in our robot. Fig.7 shows the speed of the robot over a range of contraction speeds ($6 < \dot{l} < 16$). Again, our model does an excellent job of predicting the forward motion on a horizontal surface.

The limits of our model are illustrated by its substantial difficulty predicting snake speed in a few important cases. First, our model is unable to predict horizontal speeds if the robot scales are kept at rest. Under the contraction speeds tested ($6 < \dot{l} < 16$), the robot with its scales kept flat ($\alpha = 30°$, μ_b/μ_f is 1.4) would not move forward. In contrast, our model predicts a speed of 0.12. The poor performance of the robot with scales at rest demonstrates that active control of scales are necessary for locomotion in the regime of speeds tested.

Our model also does a poor job of predicting the motion up an incline. Fig.7 shows the robot's speeds of travel up inclines of 15 and 30°. Our model predicts a negative speed on the 15 and 30° inclines, indicating the robot cannot climb but instead slides down the incline. The mis-prediction is due to the robot's reliance on static friction, which was not accounted for in our model. The robot static friction anisotropy is greater than its dynamic friction anisotropy, enabling to climb faster than predicted using a dynamic friction model. Moreover, since Froude number is large ($Fr \sim 0.39 - 1.77$) over the range of contraction speeds tested ($6 < \dot{l} < 16$), the effect of inertia is significant compared to gravitational and frictional forces. These factors together make our dynamic friction model inadequate for predicting motion up slopes.

A picture of Scalybot emerges as one sensitive to kinematics, particularly on inclines. According to our model, to move faster, the robot should increase its rate of contraction $\dot{l}$ which in turn decreases τ and increases its center-of-mass dimensional speed, $\bar{V}L/\tau$. This can be done by increasing airflow to the pneumatic cylinder. However, speed gains are lost, particularly on inclines, when the piston moves sufficiently quickly that the stationary part of the Scalybot can no longer maintain static purchase. Thus, undesirable sliding of both segments occurs at an applied force $f > \frac{1}{2}mg(\mu_b^s \cos\theta - \sin\theta)$ where μ_b^s is the backwards static friction. In our tests, such sliding was observed at contraction speeds of $\dot{l} > 10$. At such high contraction speeds, the robot no longer maintains static friction with its stationary part, and as a result, slides down the incline. In general for both snakes and snake robots, maintenance of static friction becomes increasingly important at higher angles of inclination.

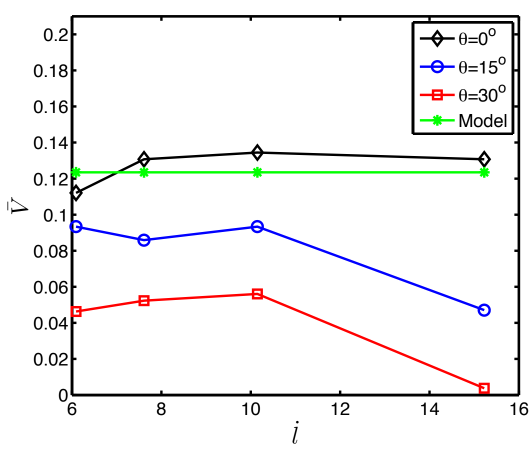

Figure 7. The velocity of center of mass, $\bar{V}$, as a function of $\dot{l}$ and inclination angle, θ. On horizontal surfaces (black lines), increasing $\dot{l}$ increases $\bar{V}$ for low contraction speeds ($\dot{l} \leq 10$). However, at sufficiently high contraction speeds ($\dot{l} > 10$) and angles of inclination (blue and red lines; $\theta \geq 15$), the velocity of Scalybot decreases due to loss of static friction anchorage.

DISCUSSION

We have modeled, designed, and constructed a simple robot that slides forward by actively modulating its belly friction. Our work was inspired by the behavior of snakes crawling up inclined surfaces, whereby they prevent backwards sliding by actively reorienting their scales. Our two-link robot uses a simple time-dependent behavior for its scales to climb slopes of 45°. This is a vast improvement in mobility: if its scales are not active, it cannot move forward even on horizontal surfaces.

Tests of Scalybot yielded insight into the subtleties of climbing not evident in our theoretical model. Specifically, we found on inclines of increasing steepness, static friction became increasingly important to prevent sliding backwards, as shown by the decreasing validity of our dynamic friction model in predicting the robot's speed. Moreover, we observed that the contraction speed has an optimum. It must be as large as possible to maximize center-of-mass speed, yet not so high as to generate forces that would break static contact of the anchored part of the robot.

To enhance its performance, we suggest the following improvements to Scalybot. More links as well as compliant joints would make Scalybot more flexible and capable of handling obstacles in its path. Snakes use transverse pushing to increase their friction force while moving through channels; adding this functionality to the robot would enable it to increase its speed and maximum inclination of climbing. Increasing the scale's range of rotation to 180° would enable Scalybot to crawl in both forward and backward directions. Moreover, adding an accelerometer to provide feedback would enable the robot to detect backward sliding and respond by digging its scales like the snakes in our experiments. Finally, by sharpening its scales, or making them compliant like that of snakes, the robot may obtain greater traction on a greater range of topographies. The scales of a corn snake are highly tapered, with a variation in thickness from 450 μm at the base to only 45 μm at the tip.

Our reported robot weight was only 1.16 kg, which did not account for a power supply or supply of pressurized air. A self-contained Scalybot housing these items would be considerably heavier. The motors powering the belly scales would then have to be chosen so that they could still lift a heavier robot. A detailed study into the forces required by snakes to activate their belly scales and lift their bodies would be of use in this regard.

There are many other attributes and behaviors of snakes that would improve the climbing ability of snake-like robots. Snakes succeed at climbing because of their high redundancy of climbing mechanisms. They have 120-350 vertebrae [12], which can both bend and twist. This allows them to lift their bodies to decrease frictional drag when sliding forward. To prevent sliding backwards they can fold their bodies and press their flanks transversely against the sides of crevices. Finally, each of their ventral scales are activated by individual muscles, which allows them even greater potential to increase their contact area over rough topographies. In our study, we have focused on their use of scales to climb, although all the above mechanisms will be important to build a successful climbing robot.

What path planning behaviors could be incorporated into Scalybot? To climb uphill, animals are known to take diagonal paths to reduce their rate of power consumption: squirrels run in helical paths up trees and antelopes travel diagonally up slopes [13]. Previous investigators have not yet observed snakes climbing diagonally or helically. On pillars much thinner than a snake length, Jayne showed that snakes perform concertina motion by coiling; however, the motion of their center of mass is purely linear [14]. A modular snake-like robot by Choset forms a helix around a column, climbing by rolling its parts while maintaining its helical configuration [15]. For pillars fatter than a snake length, snakes simply climb linearly, sometimes deviating from straight paths to follow crevices that provide greater anchorage, as shown in Fig.1. One reason snakes do not climb diagonally is their avoidance of transverse gravitational forces which may cause transverse slipping. Thus, their preference for vertical climbing may stem from the large coefficients of dynamic backwards friction (0.79 ± 0.03) relative to transverse friction (0.38 ± 0.07), according to our measurements of corn snakes on styrofoam. Although our robot cannot perform efficient path-planning strategies like moving diagonally, such topics would be of interest to future designers of climbing snake robots.

In the long run, snake-inspired robotics may benefit from consideration of a recent study by Vincent *et al.*, which presents an elegant comparison between biological and engineering systems [16]. They show technology-based problem solving relies upon the use of energy, and in contrast, biology upon information and structure. Snakes use their perceptions (eyesight, smell, vibration sensitivity, and infrared sensitivity) to gather information from their surroundings which clearly makes them more efficient and effective at moving on complex terrain. In this study, we also find that snakes take advantage of their flexible, anisotropic, and multi-functional surface structure. We hope that future development of snake-like robots will take into account structural factors such as the effects of scales.

Supplementary videos

The videos of Scalybot can be found at the following addresses or by request from the authors.

`youtube.com/watch?v=lTMfD_uOlXA`
`youtube.com/watch?v=HZyWNn4ou2A`
`youtube.com/watch?v=--LkMsDfzls`

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